Bounded-velocity Approximations of the Mobile Euclidean 2-centre

extended abstract

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1 The Euclidean 2-centre

A Euclidean 2-centre of a finite and nonempty set of points P in $\mathbb{R}^2$ is a set of two points, denoted $\Xi(P) = \{\xi_1(P), \xi_2(P)\}$, that minimizes the maximum Euclidean (ℓ_2) distance from any point p in P to the point $\xi_i(P) \in \Xi(P)$ nearest to p. Equivalently, $\xi_1(P)$ and $\xi_2(P)$ correspond to the centres of two circles whose union contains the points of P, such that the radius of the larger circle is minimized. Traditionally, the points of P represent positions of clients while the points of $\Xi(P)$ represent positions of facilities serving these clients. Given client positions, the problem is to select facility positions that minimize a given objective function representing cost as a function of distances between clients and facilities. For the k-centre, the optimization function is the maximum Euclidean distance from any client to the nearest facility.

Several efficient algorithms exist for solving the Euclidean 2-centre problem in $\mathbb{R}^2$. Eppstein [Epp92] gives algorithms that run in $O(n^2 \log^2 n \log^2 \log n)$ expected time and $O(n^2 \log^4 n)$ worst-case time. The worst-case time is improved to $O(n^2 \log n)$ using the algorithm of Jaromczyk and Kowaluk [JK94]. Sharir [Sha97] reduces the time to $O(n \log^9 n)$. Eppstein [Epp97] gives a simpler randomized algorithm in $O(n \log^2 n)$ expected time. Finally, Chan [Cha99] gives a deterministic algorithm in $O(n \log^2 n \log^2 \log n)$ time. The general Euclidean k-centre problem in $\mathbb{R}^2$ is NP-hard when k is an input parameter [MS84].

2 Mobile Clients and Facilities

Recently, motivated in part by applications in mobile computing, there has been considerable interest in recasting a number of basic questions of facility location in a mobile context [AdBG+05, AGG02, AGHV01, AH01, BBKS00, BBKS05, DK05a, DK05b, GGH+03, Her05]. Given a set of mobile clients, modelled as points in $\mathbb{R}^2$ that change continuously and with bounded velocity, the utility of a mobile facility is determined by its approximation of the optimization function as well as the continuity and maximum relative velocity of its motion. In many cases, the optimal location for a facility exhibits unbounded velocity or discontinuous motion; thus, we seek to identify functions that define the positions of mobile facilities under the dual objectives of requiring that their motion be continuous and have bounded velocity while also maintaining a good approximation of the optimization function. Closely related to the mobile Euclidean 2-centre is recent work of Bereg et al. [BBKS00, BBKS05] and Durocher and Kirkpatrick [DK05b, DK05a] that examines bounded-velocity (hence, continuous) approximations to the mobile Euclidean 1-centre and to the mobile Euclidean 1-median, both in $\mathbb{R}^2$.

3 The Mobile Euclidean 2-centre in $\mathbb{R}$ and $\mathbb{R}^2$

We show that the Euclidean 2-centre in $\mathbb{R}$ moves continuously and with relative velocity at most one. We give an algorithm for efficient maintenance of the mobile 2-centre in $\mathbb{R}$ using the kinetic data structures of Agarwal et al. [AH01, AGHV01] to maintain the extent of point sets.

By a four-client example, we demonstrate that there exist sets of mobile clients P such that every mobile Euclidean 2-centre of P moves discontinuously. Any pair of mobile facilities that move continuously must,

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Table 1: comparing approximations of the mobile Euclidean 2-centre

reflection across	guaranteed λ-approximation	lower bound on worst-case λ	relative velocity
Euclidean 1-centre	4	4	∞
rectilinear 1-centre	$2\sqrt{2} \approx 2.8284$	$2\sqrt{2} \approx 2.8284$	$2\sqrt{2}+1 \approx 3.8284$
Steiner centre	$\sqrt{10(2-\sqrt{2})} \approx 2.4203$	$2\sqrt{1+1/\pi^2} \approx 2.0989$	$8/\pi + 1 \approx 3.5465$
any mobile point		2	≥ 3

therefore, differ from the Euclidean 2-centre for some client configurations. When this occurs, the distance from some client to the nearest facility must exceed the optimal value. Let $\Upsilon(P) = \{\upsilon_1(P), \upsilon_2(P)\}$ denote a mobile facility pair, where $\upsilon_i : \mathscr{P}(\mathbb{R}^2) \to \mathbb{R}^2$. We say that Υ is a **λ-approximation** of the Euclidean 2-centre if

$$\forall P\ \forall t,\ \max_{p \in P} \min_{i \in \{1,2\}} ||p(t) - \upsilon_i(P(t))|| \leq \lambda \cdot \max_{p \in P} \min_{i \in \{1,2\}} ||p(t) - \xi_i(P(t))||.$$

We show that no mobile facility pair with maximum relative velocity less than two can guarantee a λ-approximation for any fixed $\lambda > 0$.

4 Defining Mobile Facilities by Reflection

Typically, a 2-centre problem involves partitioning the clients into two sets and subsequently identifying a center for each partition. Discontinuities in the position of a mobile 2-centre can occur when the partitions change discontinuously. To prevent this from occuring, we identify a mobile point, denoted r, that remains "central" to P while moving under bounded velocity. A client of P, p_0, is selected arbitrarily and the position of the first facility is set to coincide with that of p_0. The position of the second facility is found by reflecting p_0 across r. As natural candidates for r, we select bounded-velocity approximations of the mobile Euclidean 1-centre. These include the mobile rectilinear 1-centre [BBKS00, BBKS05] and the mobile Steiner centre [DK05b]. For comparison, we also examine the case when r is the mobile Euclidean 1-centre [BBKS00, BBKS05].

If r moves with relative velocity at most v, then the reflection of p_0 across r moves with relative velocity at most $2v + 1$. As shown by Bespamyatnikh and Kirkpatrick [BBKS00], the rectilinear 1-centre moves with relative velocity at most $\sqrt{2}$, whereas the velocity of the Euclidean 1-centre is unbounded. As shown by Durocher and Kirkpatrick [DK05b], the relative velocity of the Steiner centre is at most $4/\pi$. All three of these velocity bounds are tight, inducing the relative velocities in Tab. 1.

For facilities defined by reflection across the Euclidean 1-centre and across the rectilinear 1-centre, we show tight bounds on the λ-approximation of 4 and $2\sqrt{2}$, respectively. For facilities defined by reflection across the Steiner centre, we show $2\sqrt{1+1/\pi^2} \leq \lambda \leq \sqrt{10(2-\sqrt{2})}$. See Tab. 1.

Finally, we show that no bounded-velocity λ-approximation of the Euclidean 3-centre exists in $\mathbb{R}$.

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