

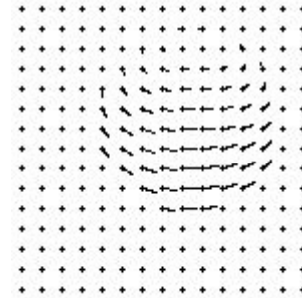
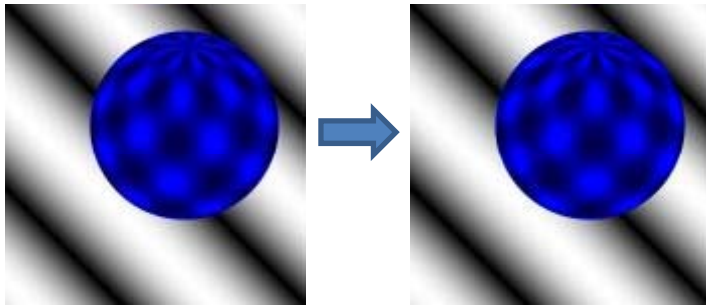
# Optical Flow

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- The perceived movements (flow) of objects in a scene.

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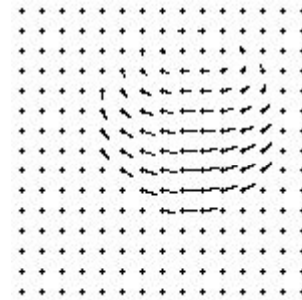
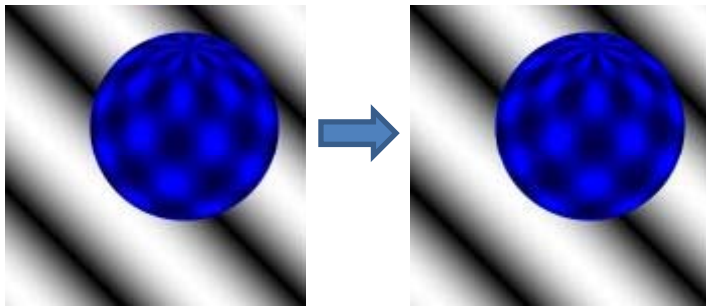
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<http://of-eval.sourceforge.net/>

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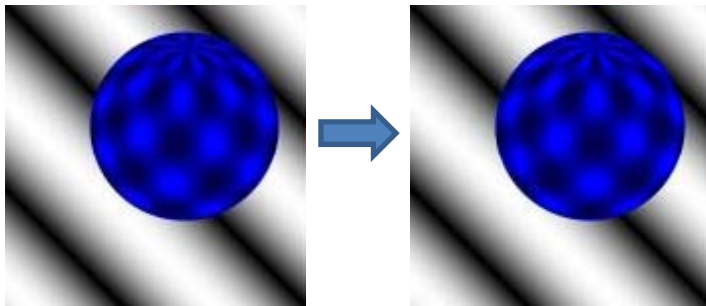
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— Length of vectors?

- speed

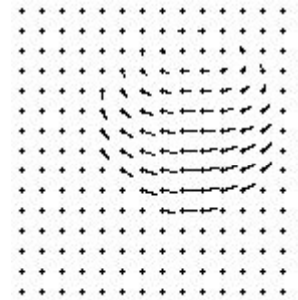
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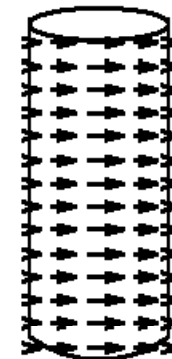


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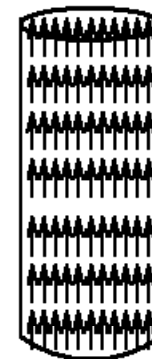
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- Vs Motion field?
- Lighting changes..



Barber's pole



Motion field



Optical flow

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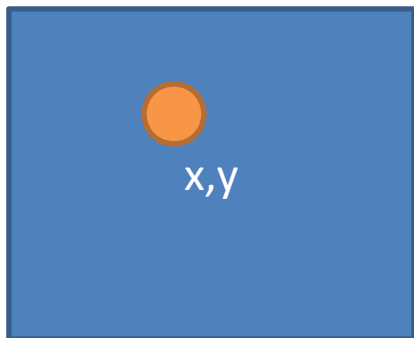
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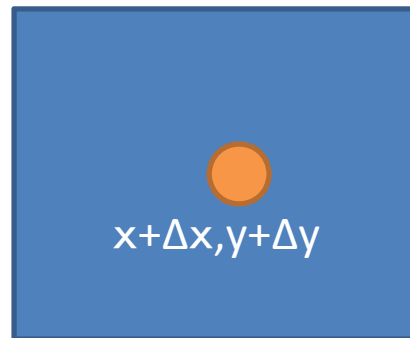
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Remember velocity vectors  $V = (u, v)$  ? Well,  $u = \frac{dx}{dt}, v = \frac{dy}{dt}$

# Constraint equation...

$$-\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x} \frac{dx}{dt} + \frac{\partial E}{\partial y} \frac{dy}{dt},$$

or

$$-\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x} u + \frac{\partial E}{\partial y} v,$$

- Solve for u,v across the image.

$\frac{\partial E}{\partial t}, \frac{\partial E}{\partial x}, \frac{\partial E}{\partial y}$  can be directly solved given  
subsequent frames

One equation per pixel, but two unknowns.....